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A Brief Talk About the Mathematical Problems of Trigonometric Functions in the National College Entrance Examination

-- Based on the Last Ten Years of NCEE Math Paper I

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Abstract

Trigonometric functions have always been an important question types in NCEE math test,as well as a hot one, and the questions appear in various forms. Its difficulty is mainly reflected in the flexible formula transformation form, which examines the students' comprehensive ability. In the multiple-choice questions, fill-in-the-blank questions are usually the finale. So in order to make the students learn to master this kind of problem better, we summarize the trigonometric function problems in the NCEE math test.

Keywords

Trigonometric functions; National college entrance examination; Mathematics; Frequently tested questions.

1. Follow Its Roots, Clarify Its Status and Basic Question Types

The mathematics examinations in trigonometry is a mandatory annual exam, the questions are generally set for one problem of multiple-choice questions (or fill-in-the-blank) and the other one of problems solving, weights 15% of the totall scores, but it's not too difficult, for the medium difficulty. The college entrance examination for trigonometric functions are mainly in the following areas: first, the image and properties of trigonometric functions; second, the simple transformation of the image of trigonometric functions; third, the constant transformation of trigonometric functions; fourth, apply trigonometric functions to solve the problems of the maximum and the minimum; fifth, the problems of trigonometric functions in the triangle.

2. Common Problem and Analysis o Trigonometric Functions in the NCEE Math Test

2.1. Basic Images and Properties of Trigonometric Functions

This kind of problem is an important test point of trigonometric function small question type, the investigation form mainly consists of the type of the combination of number and shape, and the type of the basic properties of trigonometric functions questions. The core of solving trigonometric function image type questions is based on the trigonometric function given in the question to draw out the image, including questions that are not conventional images, the tips to solve these questions are usually to find special points (zero, the maximum or the minimum value points), and combined with the parity and symmetry of trigonometric functions.

Example 1.(2019 NCEE paper I)About the function $f(x)=\sin|x|+|\sin x|$,there are the following four conclusions:

$f(x)$ is an even function; ② $f(x)$ is monotonically increasing in the interval [π/2, π]; ③ $f(x)$ has 4 zeros in [-π, π] ③ the maximum value of $f(x)$ is 2

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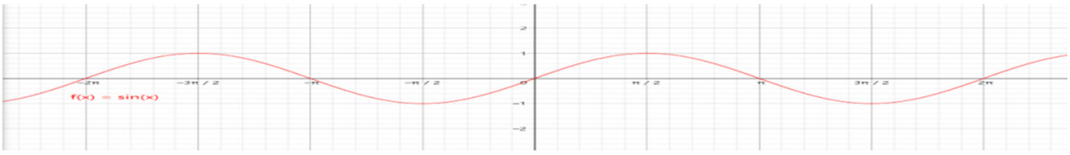
the number of the correct conclusions are:

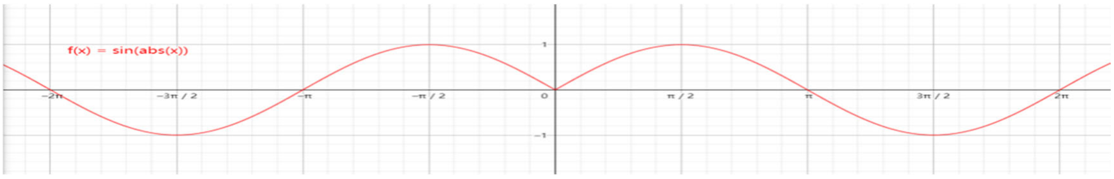
A. ①②④ B. ②④ C. ①④ D. ①③

Analysis: The question mainly investigates the properties of trigonometric functions, including parity, symmetry, monotonicity and other issues

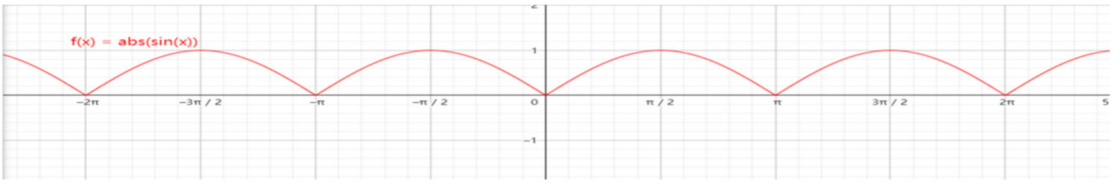
Analysis: the image of $\sin|x|$ is :



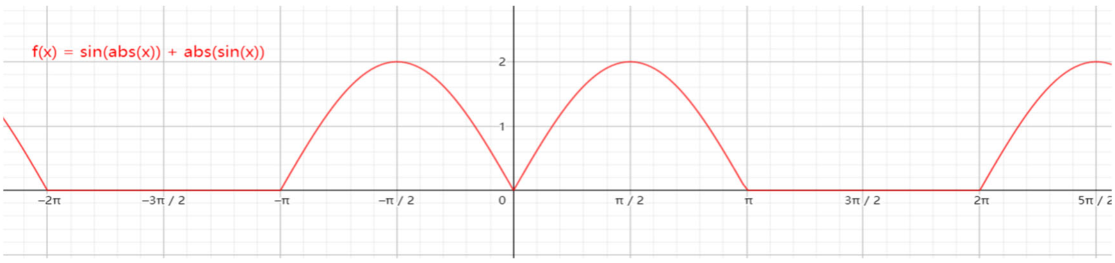
the image of $|\sin x|$ is :



The image of $\sin|x|$ is:



The image of $\sin|x|+|\sin x|$ is:



From the above images, it can be seen that:

$f(x)$ is symmetric about the y-axis, so $f(x)$ is an even function; $f(x)$ $f(x)$ is monotonically decreasing in the interval [π/2, π].

$f(x)$ has 3 zeros in [-π, π]; The maximum value of $f(x)$ is 2.

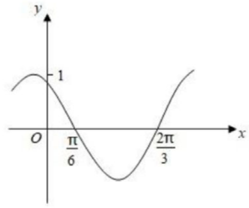
So the correct answer is ①④

2.2. Transformation of Trigonometric Function Images

There are two main ways of examining such problems:①simple transformations of images (translation, stretching);② to make it more difficult, combine the basic properties of trigonometric functions with image transformations

Example 2.(2020 new NCEE Paper I) The graph is a partial image of the function y=$sin(\omega x+\varphi)$, the whole image of $sin(\omega x+\varphi)$=()

A. $sin(x+\pi/3)$ B. $sin\left(\frac{\pi}{3}-2x\right)$ C. $cos(2x+\pi/6)$ D. $cos\left(\frac{5\pi}{6}-2x\right)$



Analysis: From the image, we know that the period of the function T=2×($\frac{2\pi}{3}-\frac{\pi}{6}$) =π, then we know $\frac{2\pi}{\omega}=\pi$, so ω=2;

From the five-point correspondence method, we get 2×$\frac{\pi}{6}$+ψ=π,thenψ=$\frac{2\pi}{3}$, so y=$sin\left(2x+\frac{2\pi}{3}\right)$=$cos\left(\frac{\pi}{2}-2x-\frac{2\pi}{3}\right)$=$cos\left(-2x-\frac{\pi}{6}\right)=cos\left(2x+\frac{\pi}{6}\right)$=$sin\left(\frac{\pi}{2}-2x-\frac{\pi}{6}\right)$= $sin\left(\frac{\pi}{3}-2x\right)$

Therefore, the answer is BC

2.3. Constant Transformation of Trigonometric Functions

Students are usually examined for their ability to simplify, calculate and evaluate trigonometric functions by constant transformations, and to test their ability to transform and convert trigonometric formulas as well as their arithmetic skills.

Example 3.(2015 NCEE paper I)$sin\,20^0\,cos\,10^0-cos\,160^0\,sin\,10^0=()$

Analysis: The problem examines the application of the formula for the sum and difference of two angles

Analysis:

$sin\,20^0\,cos\,10^0-cos\,160^0\,sin\,10^0 = sin\,20^0\,cos\,10^0-cos(180°-20°)\,sin\,10^0$ =$sin\,20^0\,cos\,10^0+cos\,20^0\,sin\,10^0$=$sin(20^{°}+10°)$=sin30°=1/2

Example 4.(2020 NCEE paper I)if α∈ (0, π), and 3cos2α-8cosα=5, then sinα=()

A. $\frac{\sqrt{5}}{3}$ B.2/3 C.1/3 D. $\frac{\sqrt{5}}{9}$

Analysis: this problem examines the application of the double angle formula

Analysis: $3\cos 2\alpha-8\cos a=5$

$3(2\,cos^2\,a-1)-8\,cos\,a=5$

$3\cos^2\alpha-4\cos a-4=0$

$cos\,\alpha=2$ (rounded off) $\cos\alpha=-\frac{2}{3}$

then $sin\,\alpha=\sqrt{1-cos^2\,\alpha}=\sqrt{1-\left(-\frac{2}{3}\right)^2}=\frac{\sqrt{5}}{3}$

So the correct answer is A

2.4. Trigonometric Optimization Problems

These problems mainly use the monotonicity of trigonometric functions, boundedness (|sinx|≤1, |cosx|≤1), derivatives to find the maximum or minimum value of trigonometric functions, or use the permutation method, using variable substitution to transform the trigonometric function of the maximum or minimum value problem into the most value of algebraic functions.

Example 5 (2018 NCEE paper Ⅰ)Given the function $f(x)=2\sin x+\sin 2x$,then the minimum value of $f(x)$ is______.

Analysis: First of all, take the derivative of the function, simplify to find $f'(x)=4(\cos x+1)(\cos x-\frac{1}{2})$,

so we can determine the monotonic interval of the function, minus interval, minus interval is

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$[2k\pi-\frac{5\pi}{3},2k\pi-\frac{\pi}{3}](k\in Z)$, increasing interval is $[2k\pi-\frac{\pi}{3},2k\pi+\frac{\pi}{3}](k\in Z)$, to determine the minimum value of the function point, so as to find $\sin x=-\frac{\sqrt{3}}{2},\sin 2x=-\frac{\sqrt{3}}{2}$,substitute it to find the minimum value of the function.

Analysis: $f'(x)=2\cos x+2\cos 2x=4\cos^2 x+2\cos x-2=4(\cos x+1)(\cos x-\frac{1}{2})$,

So when $\cos x<\frac{1}{2}$ the function monotonically decreases, when $\cos x>\frac{1}{2}$ the function monotonically increases, so that the function of the decreasing interval is $[2k\pi-\frac{5\pi}{3},2k\pi-\frac{\pi}{3}](k\in Z)$, the function of the increasing interval is $[2k\pi-\frac{\pi}{3},2k\pi+\frac{\pi}{3}](k\in Z)$, so when $x=2k\pi-\frac{\pi}{3},k\in Z$, the function $f(x)$ obtains the minimum value, in the same period, $\sin x=-\frac{\sqrt{3}}{2},\sin 2x=-\frac{\sqrt{3}}{2}$, so $f(x)_{\min}=2\times(-\frac{\sqrt{3}}{2})-\frac{\sqrt{3}}{2}=-\frac{3\sqrt{3}}{2}$, so the answer is $-\frac{3\sqrt{3}}{2}$.

The question examines the use of derivatives to study the minimum value of a function. In the process of solving such questions, you should be clear about the derivative formulas of some simple functions.

2.5. Solving Triangles with Trigonometric Functions

Such questions in the college entrance examination mainly investigate the application of trigonometric functions in triangles, often appearing in the answer questions of NCEE math test, the form of investigation is mainly the use of the sine theorem, the cosine theorem, combined with the triangle area formula to solve the relationship between sides and angles in triangles and triangles in the sides and angles of mutualization and other related issues.

Example 6. (2015 NCEE paper I) It is known that a,b,c are the opposite sides of the interior angles A,B,C of △ABC, respectively.,$\sin^2 B=2\sin A\sin C$.

(1) If a=b, find cosB;

(2) Let B=90°, a=$\sqrt{2}$, find the area of △ABC.

Analysis: First, according to the sine theorem will replace the angle to the side $\sin A=\frac{a}{2R}$,$\sin B=\frac{b}{2R}$, $\sin C=\frac{c}{2R}\sin A=\frac{a}{2R}$, From the known conditions $\sin^2 B=2\sin A\sin C$, we can get b^2=2ac, and then by the Pythagorean theorem can be found a=c=$\sqrt{2}$, and finally by the area of the triangle formula S△ABC =$\frac{1}{2}ac\sin R$, the problem can be sovled.

Example 7.(2012 NCEE paperI) a,b,c is known to be the opposite side of the three interior angles A,B,C of ΔABC, and $a\cos C+\sqrt{3}a\sin C-b-c=0$.

(I) find A;

(II) If $a=2$, the area of ΔABC $\sqrt{3}$, find b,c.

Solution: (1)From the sine theorem, we have:

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$$a\cos C+\sqrt{3}a\sin C-b-c=0\Leftrightarrow \sin A\cos C-\sqrt{3}\sin A\sin C=\sin B+\sin C$$

$$\Leftrightarrow \sin A\cos C+\sqrt{3}\sin A\sin C=\sin(a+C)+\sin C$$

$$\Leftrightarrow \sqrt{3}\sin A-\cos A=1\Leftrightarrow \sin(A-30^{\circ})=\frac{1}{2}$$

$$\Leftrightarrow A-30^{\circ}=30^{\circ}\Leftrightarrow A=60^{\circ}$$

(2) $S=\frac{1}{2}bc\sin A=\sqrt{3}\Leftrightarrow bc=4$

$$a^{2}=b^{2}+c^{2}-2bc\cos A\Leftrightarrow b+c=4$$

then b=c=2

3. Summary

Although the trigonometric function is a mandatory question in the college entrance examination in mathematics, but its difficulty is not too great, easy to score. Therefore, we should practice the problem to pragmatic basis, such as mastering the concept of trigonometric function properties and images, the conversion of various formulas, etc.. In addition to the basic solution ideas, we also need to learn the application of some mathematical ideas, such as ① the combination of number and shape: in solving the problem of the maximum or the minimum value of trigonometric function , zero point problem is generally easier to be solved with the idea of the combination of number and shape; ② the idea of transformation and transformation: in the application of trigonometric functions between the various types of formulae transformation, in the use of the sine theorem cosine theorem side angle mutualization, are required to have a strong idea of transformation.

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