

Construction of Evaluation Model of High School Mathematics Intuitive Imagination Literacy

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Abstract

Intuitive imagination literacy is one of the six essential core literacies of mathematics for students. The mathematics curriculum reform in Senior High School Guided by the development of students' subject core literacy is advancing steadily, and evaluation is the basis and guarantee for the implementation of core literacy orientation. This article is based on the process dimension of Pisa mathematical evaluation model, Under the guidance of the new curriculum standard, we should build a clearer and more perfect framework for assessing the level of intuitive imagination literacy at three standards and four levels, trying to test students' mathematical literacy from a broader perspective, in order to provide reference for the evaluation of high school students' intuitive imagination literacy.

Keywords

Intuitive imagination literacy; Evaluation framework; High school mathematics.

1. Intuitive Imagination Literacy Index System and Level Division

Evaluation is the breakthrough of curriculum reform. With the promotion of a new round of mathematics curriculum reform in the general high school, the evaluation research based on mathematics core literacy is imminent. Intuitive imagination is one of the six core qualities of mathematics proposed to the Mathematics Curriculum Standard for Senior High School (2017). It is an important means to discover and raise problems, analyze and solve problems, It is the thinking basis of exploring and forming argumentation ideas, conducting mathematical reasoning and constructing abstract structures.

Through literature review, we can find that the existing research on intuitive imagination mainly focuses on four aspects of intuitive imagination. First, the connotation and structure of intuitive imagination. For example, Feng Jing divides intuitive imagination literacy into three levels and nine stages based on information processing theory and van Hill's geometric thinking level. [1] The second is the strategy training of intuitive imagination, for example, Yin denies combined with students' own characteristics to propose a gradual and scientific training; Focus on inquiry, problem driven. [2] The third is the evaluation construction of intuitive imagination. For example, Zheng Xuejing divides the evaluation of intuitive imagination literacy into three types according to the complexity of the situation and the form of graphic presentation, and constructs the evaluation framework of intuitive imagination literacy. [3] The fourth is the analysis of intuitive imagination examination questions. For example, Li Changyi analyzed the examination of intuitive imagination literacy in the mathematics papers of the college entrance examination in 2019, It is suggested that we should pay attention to broaden the way, time and space of visual imagination teaching, strengthen the guidance of intuitive imagination strategies and methods, and strengthen the practice and perception of students' intuitive imagination [4]. It can be seen that the research on the evaluation of intuitive imagination has attracted the attention to scholars.

Therefore, this paper attempts to use the Pisa evaluation theory in the process dimension to divide the intuitive imagination mathematical process of expression mathematics, application mathematics and explanation mathematics, constructs a more perfect, clearer three- standards intuitive imagination literacy level evaluation framework, and provides reference based on the actual topic. This paper tries to test students' mathematical literacy from a broader perspective.

2. The Connotation of Intuitive Imagination Literacy and the Current Situation of Research on Evaluation

"Intuitive imagination", the core literacy of mathematics discipline proposed to the curriculum standard (2017), is the inclusion and development of the core concepts "spaces concept" and "geometric intuition" in the curriculum standard (2011), and gives a detailed explanation: "intuitive imagination refers to the literacy of using geometric intuition and spatial imagination to perceive the forms and changes of things, and using spatial forms, especially shapes, to understand and solve mathematical problems. It mainly includes: understanding the positional relationship, shape change and movement law of things by means of spatial forms; Use graphics to describe and analyze mathematical; Establish the connection between shapes and numberS, build an intuitive model of mathematical problems, and explore ways to solve problems." [5]

Using the evaluation model to analyze students' intuitive imagination literacy is a relatively large part of its research. Among them, Zheng Xuejing's evaluation research on intuitive imagination takes high school functions, geometric algebra, probability and statistics as the main content line, and constructs a five-level evaluation framework of the complexity of the situation and the presentation form of graphics. Through the analysis of students' test papers, this paper puts forward a series of reflections on the development of collaborative intuitive imagination and logical reasoning. Yin Dianyu constructed a three-dimensional "pyramid" evaluation framework of students' mathematical intuitive imagination literacy from the three dimensions of content, structure and process. Through the evaluation framework and mathematical test, This paper analyzes the level and characteristics of the core literacy of high school students' mathematical intuitive imagination, and puts forward some suggestions, such as advocating mathematical inquiry learning and following the cultivation principle of step by step. [6]

Li Rong builds an evaluation framework for intuitive imagination from two dimensions of ability and level. The ability dimension is divided into three aspects: combination of numbers and shapes, geometric intuition and spatial imagination. The content only involve the two main lines of function, geometry and algebra. It is proposed that teaching materials should be excavated and foundations should be emphasized. Step by step, suggestions for scientific training. [7]

With the help of PISA capability evaluation framework commonly used internationally to evaluate mathematical literacy or mathematical ability. Wang Gege evaluates the level of intuitive imagination literacy from three dimensions: content, process and situation, so as to establish a first-class index. It is pointed out that the core goal of geometry teaching should be to cultivate high school students' intuitive imagination quality, pay attention to geometric model thinking, develop geometric intuition and spatial imagination ability, pay attention to the transformation of three languages of mathematics, strengthen the thought of combining numbers with shapes, and pay attention to the integration of modern educational technology into classroom teaching. [8]

3. Intuitive Imagination Literacy Index System and Level Division

"The reform of mathematics curriculum in high schools oriented by developing students' subject core literacy is progressing steadily, and evaluation is the basis and guarantee for implementing the core literacy orientation." Literacy doesn't have physical attributes and can't be measured directly, but it can construct a measurable and operable index system and evaluation framework. The evaluation framework is a system structure formed by extracting to test methods of specific fields in the process of test development.

Curriculum Standard (2017) divides intuitive imagination literacy into three levels from four dimensions: situation and problems, knowledge and skills, thinking and expression, communication and reflection. However, there are some problems, such as insufficient division precision and difficulty in practical teaching. The three levels correspond to the mathematical core literacy requirements of high school graduation, college entrance examination and independent enrollment in colleges and universities. They have little guiding effect on the non-final examination, and do not take into account the situation of students that have not reached the level of high school students' intuitive imagination literacy, that is, the existence of level zero.

The framework of mathematical literacy evaluation model proposed by PISA (Program for International Students Assessment) includes three dimensions: content, process and situation, trying to examine students' mathematical literacy from a broader perspective. PISA evaluation theory covers a wider range, and the proposed situation dimension is similar to the requirements of curriculum standards. Therefore, this paper combines the three-level division and evaluation concept proposed to the Curriculum Standard (2017). Using PISA evaluation theory about reference, the evaluation framework of intuitive imagination is constructed, and four secondary indicators are subdivided under three primary indicators, and math problems with corresponding difficulty are designed for each secondary indicator, which provides topic reference for effectively evaluating students' intuitive imagination literacy.

3.1. Research and Development of Pisa Mathematical Evaluation

PISA, the abbreviation for "programme for international student assessment", is a project planned and organized by the Organization of Economic Cooperation and Development (OECD). Its purpose is to focus on students' literacy in reading, science and mathematics, and to find out whether students have mastered the problem-solving ability and lifelong learning ability needed to fully participate in society. Its evaluation concept is forward-looking and advanced in technology. It is the most influential academic evaluation in the world. Among them, the mathematical literacy evaluation model framework proposed by PISA includes three dimensions: content, process and situation. The process dimension in PISA consists of two parts: mathematical process and mathematical ability, and it is divided into eight intertwined and integrated mathematical abilities: thinking and reasoning ability, argumentation ability, communication ability and construction ability. The ability to propose and solve problems, the ability to express, the ability to use symbolic, formal and technical languages and operations, and the ability to use help and mathematical tools. PISA points out that not all of these eight abilities are applied when students use mathematics to solve problems with different situations, and most problems only need one mathematical process. Therefore, PISA mathematical literacy assessment takes the steps of students to solve mathematical problems as the dividing standard, and divides the mathematical process of: expressing mathematics, applying mathematics and explaining mathematics. According to the three mathematical processes put forward by PISA, this paper also constructs the evaluation framework of intuitive imagination literacy by combining the three levels division and evaluation concept of intuitive imagination literacy in Curriculum Standard (2017):

Table 1. Visual imagination literacy evaluation framework

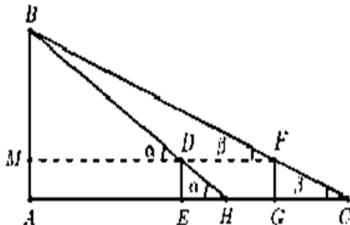
Core Literacy	Primary frame	Secondary frame
Intuitive imagination	Expressive mathematics	Level 1: Can't operate the physical geometry or identify the geometry in familiar situations.
		Level 2: Be able to establish geometric figures of objects of familiar situations, and be able to establish the connection between simple figures and objects; Experience the relationship between figure and figure, figure and quantity.
		Level 3: Be able to imagine and construct the corresponding geometric figure of the relevant situation; With the help of graphics, mathematical problems are raised, the relationship between graphics and graphics, graphics and quantity is found, and the motion law of graphics is explored.
		Level 4: Be able to put forward mathematical problems with intuitive imagination with the help of graphics in comprehensive situations.
	Applied mathematics	Level 1: Geometry figure cannot be created in the associated context.
		Level 2: Be able to discover mathematical laws by means of the nature and transformation of graphics (translation, symmetry and rotation) in familiar mathematical situations; Can describe the positional relationship, measurement relationship and unique properties of simple graphics.
		Level 3: Be able to master the basic methods of studying the relationship between figures and figures, figures and quantities, and be able to explore mathematical laws and solve practical or mathematical problems with the help of figures' properties.
		Level 4: Be able to make comprehensive use of the relationship between graph and graph, graph and quantity, and understand the relationship between various branches of mathematics; Can establish the connection between mathematics and other disciplines by intuitive imagination, and form an intuitive model of theoretical system.
	Explanatory mathematics	Level 1: Can't create geometric figures by imagination in a comprehensive situation.
		Level 2: Be able to intuitively understand mathematical problems with graphics; Can use graphics to describe and express familiar mathematical problems, enlighten ideas to solve these problems, and experience the combination of numbers and shapes. Be able to communicate visually with graphics in daily life.
		Level 3: Being able to ask mathematical questions about intuitive imagination; Be able to explore the idea of solving problems with graphics; Can form the idea of the combination of numbers and shapes, and realize the function and significance of geometric intuition. In the process of communication, can use intuitive imagination to discuss mathematical problems.
		Level 4: Be able to make comprehensive use of the relationship between graphics and figures, graphics and quantities, and understand the relationship between various branches of mathematics; With the aid of intuitive imagination, can establish the connection between mathematics and other disciplines, and form the intuitive model of theoretical system. In the process of communication, can use intuitive imagination to explore the nature of the problem and its relationship between mathematics.

4. Analysis of the Test Questions of Visual Imagination Literacy Level

In order to evaluate students' intuitive imagination literacy, this paper gives the analysis process of the evaluation tests based on a practical example of reference. Taking the evaluation

framework of Table 1 as the standard, this paper focuses on the ability and consciousness of the combination of numbers and shapes, geometry intuition and spatial imagination, and whether the students can actively use intuitive imagination to think.

Table 2. Analysis of visual imagination literacy evaluation test questions

Test Questions	Item Analysis
<p>In the Wei and Jin Dynasties, Liu Hui wrote the "island Suan Jing" which is a mathematical work about surveying. The first topic is to measure the height of the island. As shown in the figure, points e, h, G are on the horizontal line AC, DE and FG are the heights of two measuring posts which are perpendicular to the horizontal and of equal height, called "surface height", EG is called "surface distance", GC and EH are called "surface eye distance", and the difference between GC and EH is called "difference of surface eye distance". Seeking high AB of island?</p>	<p>Level 1: The geometric situation cannot be established by imagination</p>
	<p>Level 2: Be able to establish geometric situation with the help of situation, and be able to intuitively understand mathematical problems through graphics, make correct auxiliary lines, connect DF and cross AB to M, that is, $AB = AM + BM$, $\angle BDM = \alpha$, $\angle BFM = \beta$, then $\frac{MB}{\tan\beta} - \frac{MB}{\tan\alpha} = MF - MD = DF$.</p>
	<p>Level 3: On the basis of level 2, the relationship between graph and figure, figure and quantity can be found in the context of association. Contact the tangent values between the same angles, $\tan\beta = \frac{FG}{GC}$, $\tan\alpha = \frac{ED}{EH}$. So, $\frac{MB}{\tan\beta} - \frac{MB}{\tan\alpha} = MB \left(\frac{1}{\tan\beta} - \frac{1}{\tan\alpha} \right) = MB \cdot \left(\frac{GC}{FG} - \frac{EH}{ED} \right) = MB \cdot \left(\frac{GC - EH}{ED} \right)$.</p>
	<p>Level 4: On the basis of level 3, it can comprehensively utilize the relationship between graph and graph, graph and quantity, and form an intuitive model of theoretical system. According to the relationship given by the title, the MB value is obtained by simplification. $MB = \frac{ED \cdot DF}{GC - EH} = \frac{\text{Surface height} \times \text{Surface distance}}{\text{Difference of surface eye distance}}$, so, $AB = \frac{\text{Surface height} \times \text{Surface distance}}{\text{Difference of surface eye distance}} + \text{Surface height}$.</p>

5. Conclusion

In this paper, PISA mathematics evaluation as the breakthrough point, the intuitive imagination literacy of high school students evaluation index system is initially constructed, and a three-standard and four-level evaluation system is obtained. On this basis, in order to make readers clearly understand how to use it, the paper takes examples as the basis to divide and analyze, and tries to give a reasonable and applicable evaluation model. The evaluation of college entrance examination is only one of the evaluation methods of high school mathematics cores literacy. The cultivation and evaluation of intuitive imagination literacy should be based on the actual evaluation data, dynamically grasp the development level of high school students' intuitive imagination in teaching, and explore the implementation path of each mathematics discipline core literacy, so as to truly move from theory to practice.

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References

[1] Feng Jing, Xu Yatao, Wu Libao: Research on the Formation Mechanism of High School Students' Intuitive Imagination Literacy, Journal of Neijiang Normal University, vol.35 (2020) No.8,p.27-31.

- [2] Yin Dianyu: A survey on the level of high school students' mathematical intuitive imagination (MS,SuZhou University, China 2017).
- [3] Zheng Xuejing, Chen Qinghua, Wang Changping, Lin Jingrong: Measurement and Evaluation of High School Students' Intuitive Imagination Literacy, Journal of Mathematics Education, Vol.29 (2020)No.4,p.7-12.
- [4] Li Changguan: Research on the Mathematics Test Questions of the College Entrance Examination in 2019 from the Perspective of Intuitive Imagination, Basic education curriculum, Vol.16(2019),p.25-33.
- [5] Ministry of Education: Mathematics Curriculum Standard for Senior High School (2017) (People's education press,China 2018).
- [6] Zheng Xuejing, Chen Qinghua, Ke Yuehai:Construction of Evaluation Model of High School Students' Mathematical Intuitive Imagination Literacy, Educational research and experiment, Vol.3 (2021),p.71-78.
- [7] Li Rong: A survey of high school students' intuitive imagination literacy(MS,Guizhou Normal University, 2020).
- [8] Wang Qege: Research on the Evaluation of High School Students' Intuitive Imagination Literacy (MS,JiangXi Normal University, 2020).