

The Application of Game Theory on Process Teaching

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Abstract

This paper focuses on explaining the necessity of process teaching from the perspective of Game Theory. And then using the Stackelberg game model, it shows that process teaching can improve students' ability and teachers' sense of gain. In this situation the teachers and students can gradually reach the individual optimum simultaneously.

Keywords

Process Teaching; Optimal Strategy; Equilibrium; Stackelberg Game.

1. Introduction

The ultimate goal of teaching is to cultivate students, and at the same time, the teaching methods are measured through levels of students' cognition. Generally speaking, the keys to appraise students includes homework, weekly quizzes, monthly exams, midterm and final tests, and so on, which are considered as the most important part to evaluate students' abilities. However, on the one hand, the methods cannot master the time students invest. To make things worse, it also cannot make an objective judgement for students' abilities to find and solve problems as well as their self-learning and lifelong learning.

For teachers, they should attach importance to learning process, enrich teaching process sections and organize process-based appraisals for the whole process during the teaching plans so as to have a better command of real-time study. More important, in doing so, students' ability and performance can be reflected truly from a scientific and effective evaluation mechanism.

For students who major in Statistics, the work [1] concerns that how to improve process teaching and to orient the effect of process teaching. The authors [2] shows that, an online learning platform has been built with the name "Linear Algebra and Spatial Analytic Geometry" where teachers can perform a continuous use to assist teaching to improve students' learning initiative. In work [3] the learning model the authors proposed can enable students to actively follow teaching steps instead of the passive and confusing study. The authors [4] analyze problems based on Game Theory, establish mechanisms to discipline students so as to protect teachers and enhance teaching quality. In brief, all above articles takes process teaching and relative evaluation mechanism into consideration.

Different from above literature, this article makes an explanation about the necessity of process teaching from the perspective of Game Theory and the common goals pursued by both teachers and students and try hard to achieve the optimal state. In order to illustrate, the relevant knowledge about Game Theory will be first showed in the second part of this paper; the third part will analyze their learning attitude and provide relative optimal strategies to show the necessity of process teaching. The fourth part says that process teaching and additional assessment promote students' abilities and teachers' sense of gain simultaneously. And the last part interprets the maximum satisfaction owned by both sides through the Stackelberg Game, assessment contents and evaluation methods.

2. Relative Knowledge of Game Theory

Game refers to a series of strategies and actions both parties depend on mutually in the confrontations, among which the main factors are as follow: player, strategy and benefit.

(1) Player: the decision-making subjects of the Game. In this article, it refers to the teacher and students, in which the teacher serves as the manager and students as the managed.

(2) Strategy: an action plan and relative behavior of the players.

(3) Benefit: Some kinds of sense of gain players receive under a given strategy.

Nash Equilibrium is a state where all players cannot improve their returns unilaterally through changing strategies. In other words, based on opponent's strategies, each player has chosen the optimal one.

Stackelberg model is a dynamic Game model referring to oligopoly market. The model supposes that there are two firms in an oligopoly market where firm 1 is stronger and firm 2 is weaker. The stronger one (called as leading enterprise) takes action and then the weaker named as a follower gets about it. The behavior of the company is to choose yield. When the stronger one resorts to yield q_1 , the weaker one selects the yield q_2 after observation q_1 . Finally, the optimal q_1 is selected in the feasible region by the former leading enterprise, and then the q_2 is generated by the latter follower. When a strategy protects one side from an increase by other strategies, we can regard such balance as Stackelberg Equilibrium.

3. Analysis of Students' Learning Strategies

The same test results do not necessarily indicate all students have the same knowledge and ability. As usual, students who are serious, and put on emphasis on accumulated knowledge can get high marks in the tests. However, those who have lower attendance, depend on intensive rote-memorizing work before tests may also get high scores. Taking science subjects including math as an example, teachers will pay more attention to students' thinking methods and the ability of logic deduction in the class. All mentioned overall qualities can be developed step by step through kinds of sections such as situation creation, teachers' explanation, after-class discussion and operation.

According to students' orientation of learning attitude and behavior strategy, we mainly consider two types of students enjoying same status:

A--those who make great efforts to obtain credits and strive for higher academic performance;

B--those who study only for credits before exam, not for grades.

Students from Class A with high self-consciousness and self-discipline, whose learning goals are line with teachers' teaching goal nearly, will try their best to complete all tasks and orders assigned by teachers. But for Class B students who just study for credits, their goals are inconsistent with teachers' goal including organized teaching, cultivation of qualified talents. Based on that, this paper initiates process teaching for Class B students to enhance their study. For convenience, we make an assumption as follows,

(I) The number of Class B students is N ;

(II) Class B students should meet such requirements ruled by this course to get credit;

(III) The minimum requirements of the course assessment are calculated to the minimum cost constant $C_0 (> 0, \text{ necessary efforts to get credits})$; alternative assessment relative to this course is converted into alternative cost constant $C_1 (> 0)$;

(IV) Student benefits (equal to knowledge amount): it has positive correlation with the time of duration he attends and also can be converted into non-negative, increasing, continuous

revenue function defined as $R(t)$, where $t \in [0, \bar{t}]$ is the time of duration and \bar{t} is its upper bound;

(V) Student's net benefits: student's benefits $R(t)$ minus the lowest cost constant C_0 , i.e., $R(t) - C_0$;

(VI) Teachers' net benefits: (it is equal to sense of gain after teaching): the sum of the net benefits of all class B students is $N \cdot [R(t) - C_0]$.

From the perspective of Economics, any players will pursue non-negative maximal benefits, and there is no exception for students. Therefore, the strategic behavior of Class B students with the aim of credits can be concluded as follows:

$$S = \begin{cases} \text{the time of duration is } 0, & \text{if } R(0) \geq C_0, \\ \text{the time of duration is } t_0, & \text{if } R(0) < C_0 < R(\bar{t}), \\ \text{the time of duration is } 0, & \text{if } R(\bar{t}) \leq C_0, \end{cases}$$

where t_0 is the solution of $R(t_0) = C_0$. In detail,

(1) If $R(0) \geq C_0$, the minimum benefit $R(0)$ of class B student is not less than the lowest cost C_0 . And so the net benefit will be non-negative. That is to say, because of the low requirements of the course, Class B students do not need spend too much time in credit. That is why they have no motivation to learn hard. So, the optimum strategy is "the time of duration he attends is equal to zero."

(2) If $R(\bar{t}) \leq C_0$, the maximum benefit $R(\bar{t})$ of Class B student is no greater than the lowest cost C_0 , while the net benefit will be non-positive forever. Due to higher requirements of the course, Class B students with any of slack will not get credit. So, based on that, they become gradually negative to study. For them, the optimum strategy is as the same as the above.

Referring to two cases mentioned, whose assessment requirements are too high or too low, it is not suitable for teachers. In the next chapter, we will attach great importance to the situation " $R(0) < C_0 < R(\bar{t})$ ".

If $R(0) < C_0 < R(\bar{t})$, there exists a unique time length t_0 so that $R(t_0) = C_0$, and "the time of duration he attends is t_0 " is his optimal equilibrium strategy.

Firstly, t_0 is the optimal strategy. When $t = t_0$, the benefit of Class B students is $R(t_0)$. Class B students who meet the assessment demands, will get credit and keep in line with the value orientation. Besides, all Class B students will choose the time threshold ' t_0 ' as their optimal strategy.

Secondly, t_0 is also a steady Nash equilibrium strategy. When $t < t_0$, then $R(t) < R(t_0) = C_0$ and Class B students who want to get credits will extend their time length to " t_0 "; When $t > t_0$, then $R(t) > R(t_0) = C_0$ and Class B students with goal of credit will get it smoothly. And increasing study time will not increase benefits, hence they have no motivations to increase time length and fix it at " t_0 ". When $t = t_0$, then $R(t) = R(t_0) = C_0$ and Class B students will neither increase nor decrease their study time. The " t " is equal to " t_0 ". Therefore, no matter " t " changes, with time going by, all Class B students will choose t_0 as the time threshold and keep it constantly.

To sum up, " t_0 " is the optimal equilibrium strategy for all Class B students.

But, the net benefit of class B student is $R(t_0) - C_0 = 0$ now, and the net benefit of teacher $N \cdot [R(t_0) - C_0]$ is also zero. This is not what teachers expect. The reason for this phenomenon is that teachers hope students can be very proficient in the subject or even be able to improve themselves consciously. Except for the lowest requirement of C_0 , teachers do not have additional measures for them, so that there is a long distance between their attitude on the goal of credit and teachers' expectation. Finally, both sides have a completely inconsistent goal. Therefore, on the condition of no changes of the lowest requirement of C_0 , teachers will give priority to the process teaching, and improve other requirements C_1 related to this course to extend time length t . Through mentioned measures, their overall qualities and abilities will be promoted greatly so as to improve teachers' net benefit. The next part will make an explanation about how to achieve two goals simultaneously which can prove the necessity of process teaching under the additional demands.

4. Improving the Teaching Process of Teachers

Students as the participates and subordinates of whole teaching system, their strategic behaviors shall have external effect on other students and managers in the system. Sometime it is positive, other times it is negative.

Teachers regard all students as a whole and hope that the talents can be cultivated through the efforts of both teachers and students. It is just one aspect of their goals to assist students in obtaining credits. Therefore, the goals of teachers and Class B students are different, so are the strategies. In order to make the strategies of both sides of the game become consistent, some measures should be implemented by teachers to restrain students' behavior. Such as,

(1) Suppress negative external effects. Since students' individual strategic behaviors will have external effects on other students in the system, we can call it a positive external effect if the behavior (such as the conscious self-discipline of Class A students) is conducive to improving the learning enthusiasm of other individuals and so it is positive for whole students. Conversely, we can call it negative effect if the behavior (such as passing the exam by taking an abnormal method) is negative for students as a whole. For that reason, teachers need to put an emphasis on the external effects of individual students with suppression of their negative effects and amplification of their positive effects. It requires teachers to encourage students to exert positive external effects to improve the overall ability level under the condition that the minimum cost C_0 is constant and the greater the contribution, the greater the net benefit.

(2) Pay attention to the process teaching. Teachers guide students to value process teaching, deepen and consolidate their understanding of what they have learned through enriching the teaching sections (such as chapter tests, mathematics experiments, quality development, etc.) with scientific evaluation criteria and high curriculum requirements.

Then we will analyze the impact on students and teachers after adding the process teaching.

The cost requirement of Class B students to get credits is increased to $C_0 + C_1$ if we convert the process teaching sections into additional cost constant C_1 . It is because $R(t)$ is a monotonically increasing function with respect to time t , similar to Section III, there should have a unique time threshold $t_1 (> t_0)$ that makes $R(t_1) = C_0 + C_1$ true. Now, t_1 is the equilibrium strategy for Class B students with $R(t_1)$ as its benefit, and its net benefit $R(t_1) - C_0 = C_1$ is greater than 0. One the one hand, it meets the minimum assessment requirements to successfully get credits. On the other hand, it can also help students to improve themselves. The key point is that the net earning $N \cdot [R(t_1) - C_0] = NC_1$ of teachers is also greater than 0, so the net benefit of both students and teachers increased at the same time.

By adding process teaching part, the time threshold for Class B students has increased from t_0 to t_1 ; the earning of students has increased from $R(t_0)$ to $R(t_1)$; the net earnings of students have also increased from 0 to C_1 ; the net earnings of teachers have increased from 0 to NC_1 . Therefore, it is beneficial to both teachers and students to add the process teaching.

5. Reconstructing the Assessment Content of the Course

It can be seen from the previous section that if Class B students finish the process teaching part formulated by the teacher, they can get another equilibrium strategy t_1 that is different from t_0 . But it is an unstable equilibrium strategy because all the Class B students want is just to meet the assessment requirements C_0 to obtain credits. They have no motivation to complete the additional requirements C_1 arranged by teachers, so it is easy for them to deviate from the equilibrium strategy t_1 to a stable equilibrium strategy t_0 . In other words, t_1 is an unstable equilibrium strategy. It is just an idealized optimal equilibrium strategy that teachers expect students to adopt but t_0 is still the real optimal equilibrium strategy for Class B students.

For that reason, the optimal equilibrium strategy t_0 for Class B students is inconsistent with the teacher's expectations t_1 , which means that it is difficult to meet the additional requirements of teachers if students' goals have achieved already. However, corresponding measures should be implemented to restrict students and make the goals of teachers and students become consistent in order to meet the teacher's requirements.

We should notice that the status of teachers and students are not equal in the teaching system. Specifically, teachers play a role as managers (or leaders) while students are managed (or followers).

As a leader, teachers formulate rules and requirements (such as setting curriculum assessment requirements). Then under the rules, students as followers need to seek the best strategy based on their learning attitude (to be Class A students or Class B students). Therefore, we can make both teachers and students obtain the optimal at the same time through two-stage modeling with the help of Stackelberg model, which means, teachers' sense of gain is increased and students' academic level is improved. The specific analysis is as follows:

The first stage: We can learn from the analysis of the Section III that the optimal strategy for Class B students is to choose the time threshold t_0 based on the lowest cost constant C_0 because all Class B students reach an equilibrium state and have no motivation to change their strategy.

The second stage: we can learn from the Section IV that the equilibrium strategy of Class B students can be greater than t_0 , and that is why teachers set a new minimum cost constant as $\overline{C_0} = C_0 + C_1$ to link process teaching and course assessment. From Section III we can see that there is a unique t_1 that makes $R(t_1) = C_0 + C_1$. Now, compared with the lowest cost constant C_0 , the optimal equilibrium strategy for students becomes t_1 with $R(t_1)$ as its benefit, and its net benefit becomes $R(t_1) - C_0 = C_1$. Meanwhile, the net benefit of teachers reaches $N \cdot [R(t_1) - C_0] = NC_1$.

It should be noted that the time threshold t_1 obtained in the second stage is a stable equilibrium strategy. The key point of achieving its stability is to regard the requirements of process teaching as a necessity of course assessment instead of additional content. Based on that, teaching sections such as weekly tests, monthly tests, and chapter tests should be regarded as necessary components of the course assessment, and occupy a certain proportion of the final

assessment. It is more suitable for Class A students to get promotion through application questions, course essays, open questions, mathematical experiments, etc.

Both teachers and students can increase their net benefit by adding the process teaching with high requirements. Besides, the overall quality of students has been improved because the higher cost of credits. Most important, goals of students and teachers can be unified.

Finally, we add two common course assessment methods for reference.

First, formative evaluation. The work[5] refers to the evaluation of students' studying by teachers in the process of teaching, also known as process evaluation. It occurs in the teaching process immediately, repeatedly and dynamically. It aims to find specific problems in the teaching process, and adjust or solve them in time to pursue the best efficiency and effect.

In 1991, the American scholars Bangget-Drowns etc.[6] studied the relationship between the number of formative tests carried out by teachers in a semester and the improvement of students' learning achievement. They found that: formative tests are effective for students from scratch. The impact of academic performance is great, and the more tests are performed, the more students' academic achievement level improves. The following Table 1 makes a summary about the relationship between formative tests and final exams of over 1000 students in 17 classes who learn "Linear Algebra". Among that, there are five tests conducted for class 1 and 2 respectively and no tests for the remaining. From the data, on the one hand, it is concluded that the classes who have tests performed better in the final exam than the others without any tests. More important, the effect size (Effect size is an indicator that is independent of sample size and can objectively reflect the correlation strength between independent variables and dependent variables. Its calculation formula is : $(\text{experimental group mean} - \text{control group mean})/\text{standard deviation of control group.}$) of 0.22 also makes clear that formative tests play a great role in students' scores.

Table 1. The effect of formative tests

Class number	Whether to conduct formative tests	Number of formative tests	Average score	Standard deviation	Grand average	Grand standard deviation	Effect size
1	Yes	5	83.39	11.35	80.57	13.37	0.22
2	Yes	5	77.76	14.60			
3	No	0	81.08	11.04	77.58	13.38	0
4	No	0	78.66	11.21			
5	No	0	77.42	13.71			
6	No	0	76.20	14.28			
7	No	0	74.11	15.13			

And some focus [7] should go to such a misunderstanding shared by many teachers: they pay more attention to the process teaching where they usually record students' regular grades which will be calculated to the final scores at a higher rate. But on the contrary, it is because teacher put an emphasis on the scores and grades which is regarded as terminal evaluation that analysis and improvements could be ignored during the study process.

Secondly, performance assessment. Instead of regarding the test as the only way to evaluate and putting a top priority on scores, we should initiate some new type of assessment involving performance assessment to make it more effective and reliable to some extent. Performance assessment asks students to apply their previously acquired knowledge to complete tasks or deal with some problems in particular real or simulated situations. In some way, it can be served as a method to check students' mastery of relative knowledge and skills and develop

complicated skills such as problem solving, communication, cooperation and critical thinking. Detailed description can be seen reference [8,9].

The evaluation has a great focus on knowledge and methods and underlines the screening and selection. In modern society more attention goes to the diversified development. Based on that, we should care about performance duty closely related to real life such as open-ended topic, case analysis and some mathematics of research subjects connected with the realistic lifestyle applications whose aims are to test students' abilities of analysis and settlement of problems.

However, few concerns of educators are paid to evaluation skills, that is, they lack of systematic, meticulous, operational training. The performance evaluation still carries out concerns including unrealistic tasks, inadequate innovation, and poorly operational scoring rules, which will exert a negative impact on the validity and reliability of performance evaluation. In the context of educational fairness, we should make proper use of performance evaluation with "learn in practice" and make profound exploration and changes in practice.

Regardless of the formative evaluation or performance evaluation, both methods have its advantages and limitations. Guided by the development concept, we should adopt its strengths and avoid its shortcomings to promote educational revolution and innovation.

6. Conclusion

This article makes an explanation about the necessity of the process teaching through Game Theory. From the analysis of students' learning attitude, we know that there is a gap between the goals of teachers and students. However, process teaching, for one thing, enriches the assessment methods and makes students have a great difficulty in credit to deepen their understanding of the class. For another, it greatly reveals the value of teaching activities, and makes teachers and students reach an agreement on the optimal strategy, namely the goal.

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