

Fuzzy Image Reconstruction and Restoration Based on Matlab

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Abstract

This article through the analysis of fuzzy image factors, found that the factors affecting the definition of the photo are: aberration of the optical system, diffraction of the optical image, non-linear distortion of the imaging system, photosensitive nonlinearity of the photographic film, relative motion of the imaging process, Atmospheric turbulence effects, environmental random noise, etc. We used inverse filtering recovery, Wiener filtering recovery, and constrained least squares method respectively to process color blurred images and compared them to get the best model selection for Wiener filtering recovery: through Wiener filtering recovery and denoising processing. The combined method processes the black-and-white blurred picture to obtain the final restored picture, thus obtaining the best model of the Wiener filter. Compared with the original picture, the effect of the processed picture is more significant.

Keywords

Blur image processing, Noise reduction, Inverse filter restoration principle.

1. Introduction

Since the human eye has a visual persistence effect, when watching a moving object, each frame seen contains a motion process for a period of time (about 1/24 second), so the frame is actually blurred. But in general, every frame of a computer game is drawn in a clear static way, so you need a higher frame rate to feel smooth, otherwise it will not feel smooth enough. In order to achieve a smoother feeling at a lower frame rate, in computer vision technology, an algorithm capable of simulating a dynamic blur effect has been developed. When we give a picture of a dynamic blur, it is difficult to see the details of the landscape being photographed. We need to design a reasonable mathematical model to recover as clear a picture as possible. For the sake of simplicity, we can assume that the motion of the camera causes blurring, that is, all the landscapes in the figure move at the same speed.

2. Degenerate Model

2.1. Image Degradation Model Overview

The key issue in image restoration processing is the establishment of a degenerate model. When describing an image mathematically, its most common mathematical expression is

$$I = f(x, y, z, \lambda, t) \quad (1)$$

Such an expression can represent an active, colored stereo image. When studying a still, monochromatic, planar image, its mathematical expression is reduced to

$$I = f(x, y) \tag{2}$$

Based on such a mathematical expression, a degradation model as shown in Fig. 1 can be established. As can be seen from the model in Figure 1, a pure image $f(x, y)$ is degraded into an image $g(x, y)$ by passing a system H and additive noise $n(x, y)$.

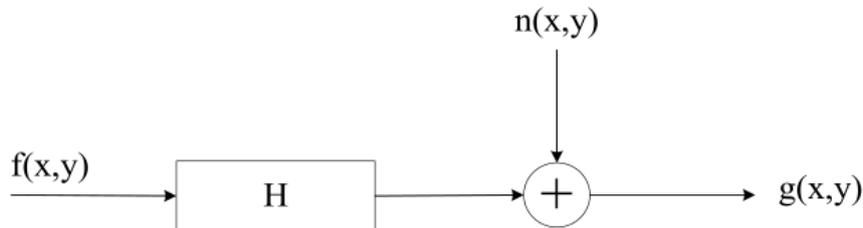


Fig 1. Image degradation model

Image restoration can be seen as an estimation process. If the degraded image $g(x, y)$ has been given and the system parameter H is estimated, $f(x, y)$ can be approximately restored. Here, $n(x, y)$ is a statistically noisy information. Of course, in order to make some optimal estimate of the processing results, a quality standard should generally be defined first. According to the degradation model of the image and the basic process of restoration, the key to the restoration process is the basic understanding of the system H . In general, a system is a unit in which certain components or components are constructed in some way. The degradation model can be divided into a continuous function degradation model and a discrete function degradation model.

2.2. Continuous Function Degradation Model

Assuming that the impulse response of system H to the impulse function $(x-\alpha, y-\beta)$ at coordinates (α, β) is $h(x, \alpha, y, \beta)$, then

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta \tag{3}$$

In the case of noise:

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta + n(x, y) \tag{4}$$

2.3. Discrete Function Degradation Model

After uniformly summing the sum, the degenerate model of the discrete function can be derived. Use one-dimensional to explain. If the sequences of $f(x)$ and $h(x)$ are respectively A and B , it is necessary to periodically extend them to a period of $M \geq A + B - 1$ in order to avoid overlapping of convolution periods.

$$f_e(x) = \begin{cases} f(x) & 0 \leq x \leq A-1 \\ 0 & A-1 \leq x \leq M-1 \end{cases} \quad h_e(x) = \begin{cases} h(x) & 0 \leq x \leq B-1 \\ 0 & B-1 < x \leq M-1 \end{cases} \tag{5}$$

Then their time domain discrete convolution can be defined as:

$$g_e(x) = \sum_{m=0}^{M-1} f_e(m)h_e(x - m) \quad x = 0, 1, \dots, M - 1 \quad (6)$$

Obviously, the above formula is also a sequence with a period M.

If a matrix is used to represent the above discrete degradation model, it can be written in the form of:

$$[g] = [H][f] \quad (7)$$

The degradation process is:

$$g_e(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_e(m, n)h_e(x - m, y - n) \quad \begin{matrix} x = 0, 1, \dots, M - 1 \\ y = 0, 1, \dots, N - 1 \end{matrix} \quad (8)$$

The image $f(x, y)$ is blurred by the linear operation $h(x, y)$ and superimposed with the noise $n(x, y)$ to form the degraded image $g(x, y)$. The degraded image is convolved with the restoration filter to obtain the restored $f(x, y)$ image as shown in Figure 2.

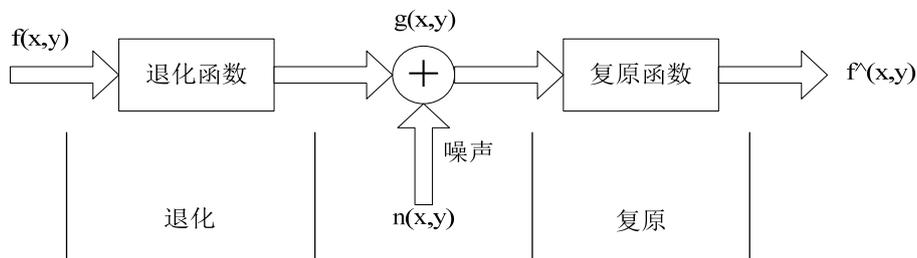


Fig 2. Image degradation/recovery process model

3. Image Restoration Technology

3.1. Inverse Filtering Recovery Principle

The easiest way to recover a degraded image is to directly inverse filter. In this method, the Fourier transform $F(u, v)$ of the degraded image is used to calculate the Fourier transform estimate of the original image, and the inverse filter degenerate formula can be obtained by the formula:

$$\hat{G}(u, v) = \frac{F(u, v) - N(u, v)}{H(u, v)} \quad (9)$$

This formula shows that inverse filtering is effective for images that are not contaminated by noise. The computational problems that may be encountered when $H(u, v)$ is close to zero at some locations in the u, v space are not considered here. Fortunately, these are ignored. Points

do not produce a sensible effect in the recovery results. However, if noise occurs, it will cause several problems: First, the influence of noise at a frequency where the magnitude of $H(u, v)$ is relatively small may become significant. This condition is usually for high frequencies u, v . In practice, the $H(u, v)$ amplitude typically decays much faster than $N(u, v)$, so the effects of noise may dominate the overall recovery. Limiting the restoration to $H(u, v)$ is large enough to get a small neighborhood at the origin of v , which can overcome this problem. The second question is about the spectrum of the noise itself. We usually don't have enough information about the noise to determine $N(u, v)$ well enough. In order to overcome the problem caused by $H(u, v)$ close to 0, add a small constant k to the denominator and modify the formula to:



Fig 3. Inverse filter recovery

3.2. Wiener Filter Restoration Principle

In most images, adjacent pixels are highly correlated, while pixels that are farther apart are less correlated. Thus, we can assume that the autocorrelation function of a typical image generally decreases as the distance from the origin increases. Since the power spectrum of the image is the Fourier transform of the autocorrelation function of the image itself, we can think that the power spectrum decreases as the frequency domain increases.

In general, noise sources tend to have a flat power spectrum, and even if this is not the case, the tendency to fall with increasing frequency is much slower than the power spectrum of a typical image. Therefore, it can be expected that the low frequency of the power spectrum is dominated by signals, while the high frequency portion is mainly occupied by noise. Since the amplitude of the inverse filter often increases as the frequency increases, the noise in the high frequency portion is enhanced. In order to overcome the above shortcomings, the method of minimum mean square error (Wiener filtering) is used for fuzzy image restoration.

Wiener filtering can be attributed to the deconvolution (or inverse filtering) algorithm, which was first proposed by Wiener and applied to one-dimensional signals, and achieved good results. Later, the algorithm was introduced into the two-dimensional signal theory, and it also achieved quite satisfactory results, especially in the field of image restoration. Because the Wiener filter has good restoration effect, low calculation amount, and excellent anti-noise performance, it is restored in the image. The field has been widely used and continuously improved, and many efficient restoration algorithms are based on this.



Fig 4. Wiener filter recovery

3.3. Constrained Least Squares Restoration Principle

Since most image recovery problems do not have a unique solution, or recovery has ill-posed features. To overcome this problem, it is often necessary to impose some constraints on the operation during the recovery process.

Let a certain linear operation Q be applied to the image, and the constraint is obtained.

$$|f - H\hat{g}|^2 = |n|^2 \tag{10}$$

Next, let \hat{g} with $|Q\hat{g}|^2$ be the smallest estimate of the original image g .

Using the Lagrangian multiplier method, first construct a helper function:

$$j(\hat{g}, \lambda) = |Q\hat{g}|^2 - \lambda(|f - H\hat{g}|^2 - |n|^2) \tag{11}$$

Solve:

$$\hat{g} = (H^T H + \gamma Q^T Q)^{-1} H^T f \tag{12}$$

We can get its frequency domain solution directly from the constrained least squares recovery of the spatial domain.

$$\hat{G}(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma |C(u,v)|^2} F(u,v) \tag{13}$$

When applying a constrained least squares recovery method, knowledge of the mean and variance of the noise is needed to give the best recovery for each given image.



Fig 5. Least squares filtering recovery

Through mat lab, this paper derives the image with the clearest picture quality in the three grades of “good”, “medium” and “poor”.

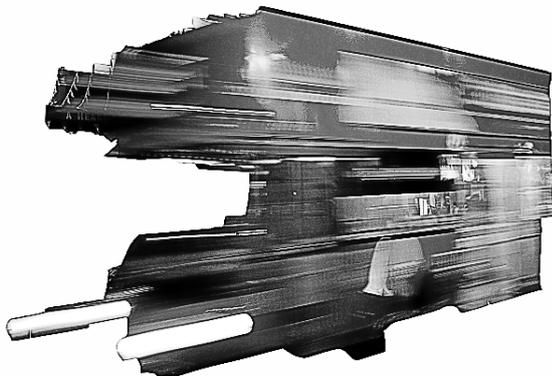


Fig 6. Processing diagram under 0.5927

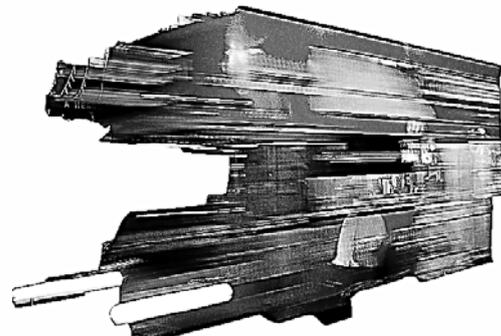


Fig 7. Processing diagram under 4.1972



Fig 8. Processing diagram under 9.5902



Fig 9. Processing diagram under 0.5927



Fig 10. Processing diagram under 4.1972



Fig 11. Processing diagram under 9.5902

4. Comparison of Recovery Results

From the restored image, the effect is still possible, because the real PSF function is used to recover, but in most cases, PSF is not known in real life, so it is necessary to analyze and then restore the image according to the specific situation.

By combining the above three methods, it can be seen that the inverse filtering, Wiener filtering, and processing effects are better by processing and comparing multiple images, and the least squares method has relatively poor processing effect. Inverse filtering mainly deals with noise-free motion blur images, which is a special case of Wiener filtering. Least squares have better restoration effects for no-noise images or low-noise images, but have a poor effect on high-noise image processing.

5. Conclusion

The methods selected in this paper are inverse filtering, Wiener filtering and constrained least squares filtering. The principles and methods of various methods are described separately, and the procedures implemented by MATLAB are given. The motion blur image is formed by three different images after PSF processing, and then processed by the above three filtering methods, and then the effects of the three methods are compared.

It can be seen from the processing results of the color picture of the model that the processing results of the inverse filtering and the Wiener filter are clearer than those of the least squares method, and the Wiener filter is a model based on the improvement of the inverse filter. The mean square error is smaller than the processing result of the inverse filter. Compared with the two, the Wiener filter is more ideal for processing color photos.

It can be seen from the processing results of black and white photos by the model that the Wiener filter is ideal for the restoration of blurred photos. From the comparison of the gray level of the image, the processed photo is similar to the original clear image, so that the processing result is in line with the human visual category.

References

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